Topological constraints on the vacuum structure of bifundamental gauge theories

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QCD with Bifundamental Quark

In this talk, I consider the $SU(n)_1 \times SU(n)_2$ gauge theory,

$$
S = \sum_{i=1}^{2} \left\{ -\frac{1}{2g_i^2} \int \mathrm{Tr}(G_i \wedge *G_i) + \frac{\mathrm{i}\theta_i}{8\pi^2} \int \mathrm{Tr}(G_i \wedge G_i) \right\}
$$
$$
+ \int \mathrm{Tr}\,\overline{\Psi}(\not{D} + m)\Psi,
$$

where G_i is the field strength of the $SU(n)_i$ gauge group,

$$
G_i = \mathrm{d}a_i + a_i \wedge a_i,
$$

and Ψ is the Dirac field in the bifundamental representation,

$$
\not{D}\Psi = \gamma^\mu(\partial_\mu \Psi + a_{1\mu}\Psi - \Psi a_{2\mu}).
$$

$SU(N_f)$-symmetric QCD with dynamical flavor gauge fields
$(N_f = N_c)$

Known Features of Bifundamental QCD

Smooth large-n limit
\# gluons $\sim$ \# fermions $\sim O(n^2)$

Orbifold daughter of $\mathcal{N} = 1$ $SU(2n)$ SYM (perturbatively)
Planar diagrams of these two theories are the same (Schmaltz, 1999; Strassler, 2001)
Nonperturbative equivalence is a difficult question due to the possible SSB of $\mathbb{Z}_2$ symmetry for orbifold projection (see, e.g., Kovtun, Ünsal, Yaffe, 2005)

Abelian confinement of deformed theory is realized on $\mathbb{R}^3 \times S^1$
Controllable semiclassical calculation can be done to explore the phase diagram (Shifman, Ünsal, 2008)

Result: Plausible phase boundaries in (θ_1, θ_2) space

Simplest possibilities for phase boundaries ($n \geq 3$) are the following:
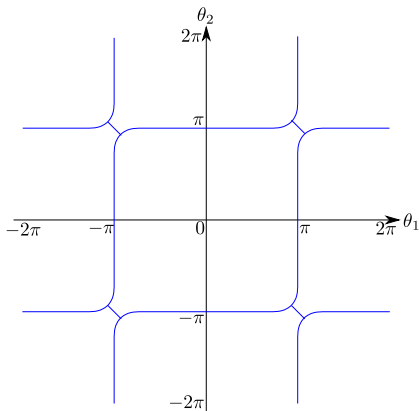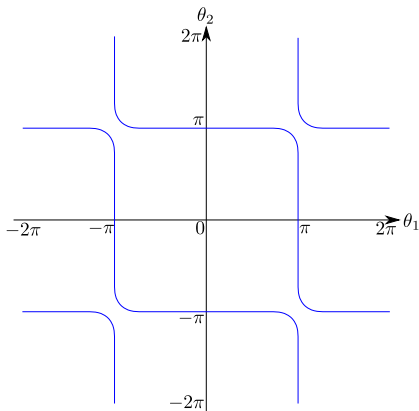
't Hooft anomaly & anomaly matching

't Hooft anomaly
= Symmetry of the quantum theory that can't be gauged

Theorem

An 't Hooft anomaly is renormalization-group invariant.

Once the 't Hooft anomaly is computed in UV, the same anomaly must be reproduced in IR ('t Hooft, 1980).

Classic Example
QCD in the chiral limit has the flavor symmetry with an 't Hooft anomaly: $SU(3)_L \times SU(3)_R \times U(1)_V$.
$\Rightarrow$ Chiral symmetry breaking and the WZW term for the pions.

Global inconsistency condition

We want to say something even when 't Hooft anomaly is absent.
$\Rightarrow$ Global inconsistency condition (Gaiotto, Kapustin, Komargodski, Seiberg, 1703.00501; Kikuchi, YT, 1705.01949, 1708.01962)

Setup

- QFT $\mathcal{T}$ with global symmetry G
- At special couplings $g = g_1, g_2$, the symmetry is larger: $G \times H$
- When gauging $G \times H$ at these points, choice of topological G-gauge theory cannot be the same

UV-IR matching

- Vacuum either at g_1 or g_2 must be nontrivial (e.g., SSB of $G \times H$) (Gaiotto, Kapustin, Komargodski, Seiberg, 1703.00501), **or**
- Theories at g_1 and g_2 are separated by phase transition (Kikuchi, YT, 1705.01949, 1708.01962)

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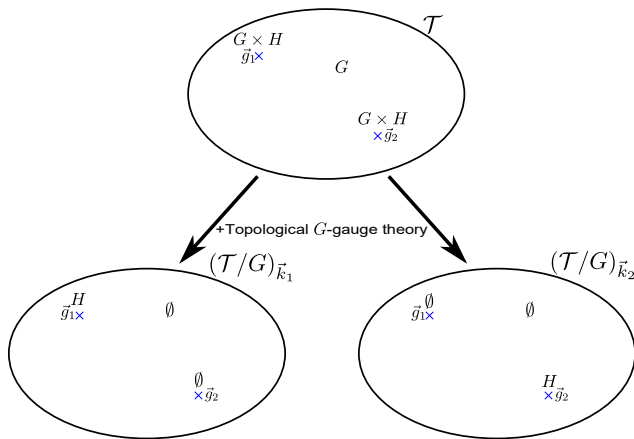
Symmetries of QCD(BF)

The massive bifundamental QCD has following global symmetries:

- There's the diagonal center $\mathbb{Z}_n$ one-form symmetry, which transforms

$$W_1(C) = \text{Tr}\left[\mathcal{P}\exp\oint_C a_1\right], \quad W_2(C) = \text{Tr}\left[\mathcal{P}\exp\oint_C a_2\right],$$

as $(\omega_n = \exp(2\pi i/n))$

$$W_1(C) \mapsto \omega_n W_1(C), \quad W_2(C) \mapsto \omega_n W_2(C).$$

- $(\mathbb{Z}_2)_{CP}$ exists only at $\theta_{1,2} = 0, \pi$.

Basic idea of the strategy

We introduce a background gauge field for the $(\mathbb{Z}_n)_{center}$ one-form symmetry,

$$Z_{(\theta_1, \theta_2), p}[B] = \int \mathcal{D}a_i \exp\left(-S[G_i + B] + \mathrm{i}\frac{np}{4\pi}\int B \wedge B\right).$$

B: $\mathbb{Z}_n$ two-form gauge field, i.e. $nB = \mathrm{d}A$.

At $(\theta_1, \theta_2) = (\ell_1\pi, \ell_2\pi)$, the original theory has the CP symmetry, so we compute

$$Z_{(\ell_1\pi, \ell_2\pi), p}[\mathsf{CP} \cdot B] = Z_{(\ell_1\pi, \ell_2\pi), p}[B] \exp\left(\mathrm{i}\mathcal{A}_{(\ell_1\pi, \ell_2\pi), p}[B]\right).$$

Anomaly and global inconsistency of QCD(BF)

The phase functional is given by

$$\mathcal{A}_{(\ell_1\pi,\ell_2\pi),p}[B] = \frac{n}{4\pi}(2p - \ell_1 - \ell_2)\int B \wedge B.$$

Thus, CP invariance with the background gauge field B requires $2p = \ell_1 + \ell_2$ modulo n.

- At $(\theta_1,\theta_2) = (\pi,0), (0,\pi)$, $(\mathbb{Z}_n)_{center}$ and CP have a mixed anomaly for even n and global inconsistency for odd n.
- High symmetry points $(\theta_1,\theta_2) = (0,0), (\pi,\pi)$ have global inconsistency but no 't Hooft anomalies.
- No global inconsistency exists between $(\theta_1,\theta_2) = (0,0), (\pi,-\pi)$.

Result: Plausible phase boundaries in (θ_1, θ_2) space

- At $(\theta_1, \theta_2) = (\pi, 0), (0, \pi)$, CP is spontaneously broken.
- CP at $(\theta_1, \theta_2) = (\pi, \pi)$ is spontaneously broken, or the phase-transition line separates $(\theta_1, \theta_2) = (0, 0), (\pi, \pi)$.

(YT, Kikuchi; 1705.01949)
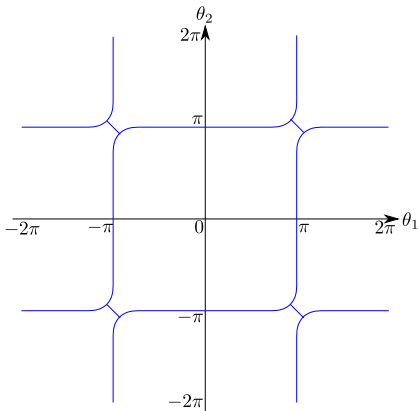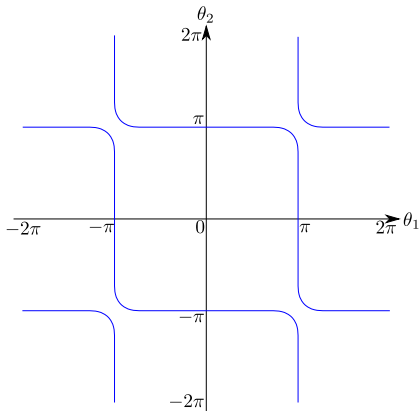
Summary

- Global inconsistency is a powerful nonperturbative tool.
- If there is an 't Hooft anomaly, the ground states are nontrivial.
- Even if there is no mixed anomaly, the global inconsistency requires a phase transition by comparing high-symmetry points:
  - Symmetry is spontaneously broken at one of high-symmetry points, or
  - High-symmetry points are separated by phase transitions.
- Bifundamental gauge theory is considered, and we put the nonperturbative constraints on the phase diagram.

Related unmentioned works: Komargodski, Sharon, Thorngren, Zhou, 1705.04786;

Komargodski, Sulejmanpasic, Ünsal, 1706.05731; Shimizu, Yonekura, 1706.06104; Gaiotto,

Komargodski, Seiberg, 1708.06806